



## INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

### IMPROVEMENT OF CLASSICAL LUMPED MODEL FOR TRANSIENT HEAT CONDUCTION IN SLAB USING HERMITE APROXIMATION

Rakesh Krishna Yadav\*, Avin Chandrakar

\* Oriental University, Indore.  
Oriental University, Indore.

#### ABSTRACT

The present work aims at applying the ideas on the analysis of improved lumped-parameter model for transient heat conduction in a slab with temperature-dependent thermal conductivity. The transient temperature is found to depend on various model parameters, namely, Biot number, heat source parameter and time. Polynomial Approximation Method (PAM) has been possible to derive a unified relation for the transient thermal behavior of solid (slab and tube) with both internal generation and boundary heat flux. In all the cases, a closed form solution is obtained between temperature, Biot number, heat source parameter and time.

An improved lumped parameter model has been adopted through two point Hermite approximations for integrals. For linearly temperature-dependent thermal conductivity, it is shown by comparison with numerical solution of the original distributed parameter model that the higher order lumped model (H1,1/H0,0 approximation) yields significant improvement of average temperature prediction over the classical lumped model. A unified Biot number limit depending on a single dimensionless parameter  $\lambda$  is given both for cooling and heating processes. The result of the present analysis has been compared with earlier numerical and analytical results. A good agreement has been obtained between the present prediction and the available results.

**KEYWORDS:** Hermite approximations, PAM, Temperature-dependent thermal conductivity, Lumped model, Nonlinear heat Conduction, Transient heat conduction, Biot number.

#### INTRODUCTION

In this chapter introduces the mechanism of heat transfer known as conduction. In the context of engineering applications, this is more likely to be representative of the behavior in solid than fluids. Conduction phenomena may be treated as either time-dependent or steady state. Time-dependent conduction has been simplified to the extreme cases of  $Bi \ll 1$  and  $Bi \gg 1$ . For the former, the lumped method may be used and in the latter the semi-infinite method. It is worth noting that in both cases these method are used in practical applications in the inverse mode to measure heat transfer coefficient from a known temperature-time history.

#### METHODOLOGY

According to Lumped Body models we first introduce the spatially averaged dimensionless temperature as follows:

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial}{\partial \eta} \left( \lambda(\theta) \frac{\partial \theta}{\partial \lambda} \right), \quad \text{in } 0 < \lambda < 1 \text{ for } \tau > 0, \quad (1)$$

$$\theta_{av}(\tau) = \int_0^1 \theta(\eta, \tau) d\eta \quad (2)$$

We operate Eq. (1) by  $\int_0^1 d\eta$ , using the definition of average temperature, Eq. (2), we get

$$\frac{d\theta_{av}(\tau)}{d\tau} = \left( \frac{\lambda(\theta)\partial\theta}{\partial\eta} \right)_{\eta=1} - \left( \frac{\lambda(\theta)\partial\theta}{\partial\eta} \right)_{\eta=0} \quad (3)$$

Now, the boundary conditions

$$\frac{d\theta_{av}(\tau)}{d\tau} = -B_i\theta(1, \tau) \quad (4)$$

Eq. (4) is an equivalent integral-differential formulation of the mathematical model with no approximation involved. Supposing that the temperature gradient is sufficiently uniform over the whole spatial solution domain, the classical lumped system analysis (CLSA) is based on the assumption that the boundary temperatures can be reasonably well approximated by the average temperature, as

$$\theta(0, \tau) \cong \theta(1, \tau) \cong \theta_{av}(\tau),$$

Which leads to the classical lumped model

$$\frac{d\theta_{av}(\tau)}{d\tau} = -B_i\theta_{av}(\tau)$$

And to be solved with the initial condition for the average temperature

$$\theta_{av}(0) = 1$$

It can be seen that the classical model shows no influence of the temperature-dependent thermal conductivity.

Alhama and Zueco identified four different kinds of problem that may occur: (i) a heating process with a positive temperature-dependent coefficient,  $k_2 > 0$ ; (ii) a heating process with  $k_2 < 0$ ; (iii) a cooling process with  $k_2 > 0$  and (iv) a cooling process with  $k_2 < 0$ . They established that the universal mean Biot number limit for applying the lumped model can be expressed as a function of the dimensionless number  $k = (k_{max} - k_{min})/k_m$ , and the kind of process (cooling or heating), with  $k_m = (k_{max} + k_{min})/2$

In proper choice of dimensionless parameters, the four kinds of problem can be reduced to two kinds of problem: (i)  $\beta > 0$ , representing cooling with a positive temperature-dependent coefficient  $b > 0$  or heating with  $b < 0$  and (ii)  $\beta < 0$ , representing cooling with  $b < 0$  or heating with  $b > 0$ . The main difference between Alhama–Zueco’s analysis and ours lies in the choice of the reference temperature. While Alhama and Zueco always use the minimum temperature  $T_{min}$  as the reference temperature, we always use the surrounding fluid temperature  $T_\infty$  as the reference temperature whether cooling or heating. For a linearly temperature dependent thermal conductivity.

$$k(T) = k_\infty\{1 + b(T - T_\infty)\}$$

We have for a cooling process ( $T_i > T_\infty$ ) with a positive temperature dependent coefficient ( $b > 0$ )

$$\lambda(\theta) = \frac{k(T)}{k_\infty} = 1 + b(T_i - T_\infty)\theta = 1 + \beta\theta$$

Thus  $\beta = b(T_i - T_\infty) > 0$ . For a cooling process with  $b < 0$ , we have

$$\lambda(\theta) = 1 + b(T_i - T_\infty)\theta = 1 + \beta\theta$$

with  $\beta = b(T_i - T_\infty) < 0$ . For a heating process ( $T_i < T_\infty$ ) with  $b > 0$ , we have

$$\lambda(\theta) = 1 + b(T_i - T_\infty)\theta = 1 + \beta\theta$$

with  $\beta = b(T_i - T_\infty) > 0$ .

It can be seen that the four kinds of problem identified by Alhama and Zueco [2] can be represented conveniently by only one dimensionless parameter  $\beta$ , with  $\beta > 0$  representing cooling with  $b > 0$  or heating with  $b < 0$ , and  $\beta < 0$  representing cooling with  $b < 0$  or heating with  $b > 0$ .

We proceed to examine the example problems given by Alhama and Zueco.

**Problem 1.**  $T_i = 1, T_\infty = 0, k(T) = 0.9+0.2T, k_{\min} = 0.9, k_{\max} = 1.1, k_m = 1, k = 0.2$

By our analysis,  $\theta = T, \theta_i = 1, \theta_\infty = 0,$   
 $\lambda(\theta) = kT/k_\infty = 1+(2/9)\theta, \beta = 2/9$

**Problem 2.**  $T_i = 1, T_\infty = 0, k(T) = 1.8+0.4T, k_{\min} = 1.8, k_{\max} = 2.2, k_m = 2, k = 0.2$

By our analysis,  $\theta = T, \theta_i = 1, \theta_\infty = 0, \lambda(\theta) = kT/k_\infty = 1+(2/9)\theta, \beta = 2/9$

**Problem 3.**  $T_i = 10, T_\infty = 0, k(T) = 0.9+0.02T, k_{\min} = 0.9, k_{\max} = 1.1, k_m = 1, k = 0.2$

By our analysis,  $\theta = T/10, \theta_i = 1, \theta_\infty = 0,$   
 $\lambda(\theta) = kT/k_\infty = 1+(2/9)\theta, \beta = 2/9$

The difference between Alhama-Zueco's and our analysis is shown when examining the heating processes with a positive temperature-dependent coefficient ( $k_2 > 0$  or  $b > 0$ ).

**Problem 4.**  $T_i = 0, T_\infty = 1, k(T) = 0.9+0.2T, k_{\min} = 0.9, k_{\max} = 1.1, k_m = 1, k = 0.2$

By our analysis,  $\theta = (T - 1)/(-1), \theta_i = 1, \theta_\infty = 0,$   
 $\lambda(\theta) = \frac{kT}{k_\infty} = \frac{0.9+0.2(-\theta+1)}{1.1} = 1 - \frac{2}{11}\theta,$   
 Thus  $\beta = 2/11$

**Problem 5.**  $T_i = 0, T_\infty = 1, k(T) = 1.8+0.4T, k_{\min} = 1.8, k_{\max} = 2.2, k_m = 2, k = 0.2$

By our analysis,  $\theta = (T - 1)/(-1), \theta_i = 1, \theta_\infty = 0,$   
 $\lambda(\theta) = \frac{kT}{k_\infty} = \frac{1.8+0.4(-\theta+1)}{2.2} = 1 - \frac{2}{11}\theta,$   
 Thus  $\beta = 2/11$

**Problem 6.**  $T_i = 0, T_\infty = 10, k(T) = 0.9+0.02T, k_{\min} = 0.9, k_{\max} = 1.1, k_m = 1, k = 0.2$

By our analysis,  $\theta = (T - 10)/(-10), \theta_i = 1, \theta_\infty = 0,$   
 $\lambda(\theta) = \frac{kT}{k_\infty} = \frac{0.9+0.02(-\theta+10)}{2.2} = 1 - \frac{2}{11}\theta,$

Thus  $\beta = 2/11$

It can be seen that problems 1-3 reduce to a same dimensionless problem with  $\beta = 2/9$  and problem 4-6 reduce to another dimensionless problem with  $\beta = -2/11$

## RESULT AND DISCUSSION

The solutions of classical and improved lumped models are shown in tabular and graphical forms in comparison with a reference finite difference solution of the original distributed model, The initial boundary value problem defined by using an implicit finite difference method, with a 201 nodes mesh in spatial discretization and a dimensionless time step of 0.00001 for all cases. Different values of the Biot number  $Bi$  and the parameter  $b$  are chosen so as to assess accuracy of the solutions given by lumped models.

In Table4.1, it is presented a comparison of the dimensionless average temperatures obtained by lumped models and the reference finite difference solution of the original distributed parameter model at different values of time, for  $Bi = 1.0$  and  $\beta = 1.0$ . As can be seen, the classical lumped model gives an error of 0.0681 at  $\tau = 1.0$ , while the  $H_{0,0}/H_{0,0}$  model gives an error of 0.0137 at  $\tau = 1.0$ , and the  $H_{1,1}/H_{0,0}$  model yields a maximum error less than 0.005 for all time values. Fig.4.1 shows the comparison of the dimensionless average temperatures for  $Bi = 2.5$  and  $\beta = 0.5$ . It can be seen

that the solution give by the higher order improved lumped model ( $H_{1,1}/H_{0,0}$ ) agrees quite well with the finite difference solution.

*Table 1 Comparison of lumped model against finite different solution average temperature  $\theta_{av}(\tau)$  at different value of time*

$\tau$	FD solution $B_i = 1.0$	CLSA $\beta = 1.0$	$H_{0,0}/H_{0,0}$	$H_{1,1}/H_{0,0}$
0.1	0.9150	0.9048	0.9157	0.9190
0.2	0.8406	0.8187	0.8389	0.8450
0.3	0.7730	0.7408	0.7689	0.7774
0.4	0.7113	0.6703	0.7050	0.7156
0.5	0.6548	0.6065	0.6466	0.6589
0.6	0.6031	0.5488	0.5934	0.6070
0.7	0.5557	0.4966	0.5447	0.5595
0.8	0.5123	0.4493	0.5002	0.5159
0.9	0.4725	0.4066	0.4595	0.4758
1.0	0.4359	0.3679	0.4222	0.4391
2.0	0.1985	0.1353	0.1838	0.1997
3.0	0.0925	0.0498	0.0813	0.0926
4.0	0.0436	0.0183	0.0363	0.0434
5.0	0.0207	0.0067	0.0163	0.0204

## CONCLUSION

The improved lumped parameter models are presented for transient heat conduction in a slab with cubically temperature-dependent thermal conductivity and subject to convective cooling or heating. Improved lumped models are obtained through two point Hermite approximations for integrals. For linearly temperature-dependent thermal conductivity, it is shown by comparison with numerical solution of the original distributed parameter model that the higher order lumped model ( $H_{1,1}/H_{0,0}$  approximation) yields significant improvement of average temperature prediction over the classical lumped model. It is shown that the maximum relative error of the dimensionless average temperature is influenced predominantly by the Biot number. A unified Biot number limit is obtained as a function of the linear dependence coefficient  $\beta$ ,  $B_{i\text{limit}} = 0.523\beta + 1.078$  for  $-0.6 \leq \beta \leq 0.8$ . The lumped model  $H_{1,1}/H_{0,0}$  is expected to yield maximum normalized error less 0.01 for  $B_i < B_{i\text{limit}}$ , for a given  $\beta$ .

## REFERENCES

- [1] F. Alhama, A. Campo, Electric network representation of the unsteady cooling of a lumped body by nonlinear heat transfer modes, *J. Heat Transf.* 124 (2002) 988–992.
- [2] F. Alhama, J. Zueco, Application of a lumped model to solids with linearly temperature-dependent thermal conductivity, *Appl. Math. Model.* 31 (2007) 302–310.
- [3] P. Keshavarz and M. Taheri, “An improved lumped analysis for transient heat conduction by using the polynomial approximation method”, *Heat Mass Transfer*, 43, (2007), 1151–1156
- [4] J. Su, Improved lumped models for asymmetric cooling of a long slab by heat convection, *Int. Commun. Heat Mass Transf.* 28 (2001) 973–983.
- [5] J. Su, Improved lumped models for transient radiative cooling of a spherical body, *Int. Commun. Heat Mass Transf.* 31 (2004) 85–94.
- [6] J. Su, R.M. Cotta, Improved lumped parameter formulation for simplified LWR thermohydraulic analysis, *Ann. Nucl. Energy* 28 (2001) 1019–1031.
- [7] N.J. Rupert, C.V. Falkenberg, R.M. Cotta, J. Su, Engineering analysis of ablative thermal protection for atmospheric reentry: improved lumped formulations and symbolic-numerical computation, *Heat Transf. Eng.* 25 (2004) 101–111.
- [8] A.C. Pontedeiro, J. Su, R.M. Cotta, Improved lumped model for transient heat conduction in a heat generating cylinder with temperature-dependent thermophysical properties, in: *Proceedings of 2004 ASME Heat Transfer/Fluids Engineering Summer Conference*, Charlotte, North Carolina, USA, July, 11–15, 2004
- [9] E.J. Correa, R.M. Cotta, Enhanced lumped-differential formulations of diffusion problems, *Appl. Math. Model.* 22 (1998) 137–152.